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Modulational interaction of short-wavelength ion-acoustic oscillations in impurity-containing plasmas

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The modulational interactions of ion acoustic waves are investigated for plasmas containing heavy impurity ions or dust particles. Low-frequency modulations arising from dust-acoustic oscillations are considered. It is found that instability of the short-wavelength ion-acoustic, or ion Langmuir, waves can develop for modulations with wavelengths larger as well as smaller than those of the pump waves. The latter possibility can result in the appearance of waves resonant with thermal ions. It follows that the bulk of the plasma ions can be heated. The corresponding one-dimensional ion Langmuir solitary waves are also discussed.

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I. INTRODUCTION

Investigation of plasmas containing heavy impurity ions or dust particles is important [1–7] for the understanding of space and astrophysical phenomena (planetary rings, cometary tails, interstellar clouds, etc.), the Earth's environment (noctilucent clouds, auroras, etc.), many laboratory and technological plasmas (low-temperature rf and dc glow discharges, rf plasma etching, the wall region fusion plasmas, etc.), as well as many materials (semiconductors, dusty crystals, etc.). The impurity ions or dust particles often have large mass, are charged negatively with large charge numbers ($|Z_d|$ up to 10^3), and are of average sizes usually much less than the Debye length [8,9]. Such charged impurity particles can significantly modify the properties of the normal modes and their evolution [10–17]. Moreover, their presence results in the appearance of new normal modes, such as the dust-acoustic waves [12,14], which involve oscillations at such low frequency that the electrons and ions remain in equilibrium and the dynamics is mainly due to the dust particles.

Nonlinear processes that can disturb the wave structure and transfer energy to or from the plasma particles are important in many situations. Studies of transitional scatterings of waves off dust particles [9,18,19] and the effect of decays and induced scatterings involving the dust-acoustic waves [20] have indicated the importance of these effects in dusty plasmas. An important nonlinear process is the modulational interaction [21–23] of finite amplitude waves, whose amplitude and phase are modulated by much lower frequency motion. It is the basic process for transition from weak to strong turbulence in plasmas. The strongly turbulent state is characterized [21–25] by strong phase correlation among the excited modes within coherent objects (such as solitons, phase-space holes, and collapsing cavities) with chaotic interaction among the latter. Modulational instabilities are the main source for these coherent objects.

In the development of the modulational instability, plasma is expelled from regions of high wave energy by the ponderomotive force, which results from the nonlinear coupling between the high-frequency waves with much lower frequency density perturbations, which are usually quasineutral. For an impurity-free plasma, the latter are usually from ion motion. The presence in dusty plasmas of the even lower frequency dust-acoustic oscillations results in the possiblility modulational instabilities associated with the latter. Such an instability for finite amplitude electromagnetic waves has been studied [15]. In this paper we investigate the modulational instability of short-wavelength ion-acoustic waves in a dusty plasma. Linearly, the waves have negative dispersion. It is found that instability with perturbation wavelengths exceeding those of the pump wave is possible. The instability can lead to resonance between the enhanced dust-acoustic oscillations and the thermal ions, resulting in the heating of the bulk plasma ions. One-dimensional ion-wave solitons which can appear in the evolution of the instability will also be discussed.

The paper is organized as follows. In Sec. II, we present the dusty plasma model and obtain the evolution equations for the slowly varying short-wavelength ion waves. In Sec. III, we obtain the dispersion relation for the modulational interaction of the pump ion waves with quasineutral low-frequency density perturbations associated with the dust-acoustic mode. In Section IV, we evaluate the dispersion relation and present the existence conditions and growth rates of the modulational instability. Section V is devoted to a localized solution of the evolution equations for the slowly varying envelope of the short-wavelength ion waves. In Sec. VI, our results are summarized and discussed.

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II. EVOLUTION EQUATIONS

Consider a uniform unmagnetized collisionless hydrogen plasma containing massive impurity particles or dust grains with an average negative charge $Z_d e$. When dust grains are involved, the size of the latter is assumed to be much smaller than the Debye length, the wavelength of the perturbations, as well as the distance between the plasma particles. Thus we can treat the dust grains as negatively charged point masses. We also assume that the following conditions are fulfilled:

$$\begin{aligned} &T_e \gg T_i \gg T_d/Z_d, \\ &m_e \ll m_i \ll m_d/Z_d, \\ &n_{0e} \sim n_{0i} \sim n_{0d}Z_d, \end{aligned} \tag{1}$$

where m_α, $n_{0\alpha}$, and T_α ($\alpha = e, i, d$) are the mass, the unperturbed density, and the temperature of the electrons, ions, and dust particles, respectively. Thus both ion-acoustic and dust-acoustic oscillations can exist in the plasma.

For $|\mathbf{k}|r_{Di} \ll 1$ the linear dispersion relation of the ion-acoustic oscillations is given by

$$\omega_{\mathbf{k}}^2 = \frac{\omega_{pi}^2|\mathbf{k}|^2 r_{De}^2}{1+|\mathbf{k}|^2 r_{De}^2} \tag{2}$$

and that for the long-wavelength ($|\mathbf{k}|r_{Di} \ll 1$) dust-acoustic waves by

$$\omega_{\mathbf{k}} = |\mathbf{k}|v_{sd}, \tag{3}$$

where $\omega_{\mathbf{k}}$ is the wave frequency, $\mathbf{k}$ is the wave vector, $\omega_{pi} = (4\pi n_{0i}e^2/m_i)^{1/2}$ is the ion plasma frequency, $r_{De(i)} = (T_{e(i)}/4\pi n_{0e(i)}e^2)^{1/2}$ is the electron (ion) Debye length, and

$$v_{sd} = \left(\frac{n_{0d}Z_d}{n_{0i}}\right)^{\frac{1}{2}} \left(\frac{Z_d T_i}{m_d}\right)^{\frac{1}{2}} \tag{4}$$

is the dust-acoustic speed.

The long-wavelength limit ($|\mathbf{k}|r_{De} \ll 1$) of (2) describes the ordinary ion-acoustic waves. In the short-wavelength limit ($|\mathbf{k}|r_{Di} \ll 1 \ll |\mathbf{k}|r_{De} \ll \sqrt{m_i/m_e}$, the last inequality being needed in order to avoid strong electron Landau damping), the dispersion relation can be written as

$$\omega_{\mathbf{k}} = \omega_{pi}\left(1 - \frac{1}{2|\mathbf{k}|^2 r_{De}^2}\right). \tag{5}$$

The dispersion relation (5) for short-wavelength ion-acoustic waves (sometimes called ion Langmuir waves) is typical for media with inverse dispersion [26].

In the earlier treatment of the modulational instability of ion Langmuir waves [26] in an impurity-free plasma, the low-frequency perturbations were associated with the long-wavelength ion-acoustic oscillations. Here we shall investigate the case in which the modulation is associated with density perturbations arising from the dust-acoustic mode. The kinetic equation describing the distribution function $f_{\mathbf{p}}^{(\alpha)}$ is

$$\frac{\partial f_{\mathbf{p}}^{(\alpha)}}{\partial t} + \mathbf{v}\cdot\frac{\partial f_{\mathbf{p}}^{(\alpha)}}{\partial \mathbf{r}} + q_\alpha \mathbf{E}\cdot\frac{\partial f_{\mathbf{p}}^{(\alpha)}}{\partial \mathbf{p}} = 0, \tag{6}$$

where $q_e \equiv e$, $q_i \equiv -e$, and $q_d \equiv Z_d e$; $f_{\mathbf{p}}^{(\alpha)}$ is normalized such that $\int f_{\mathbf{p}}^{(\alpha)} d\mathbf{p}/(2\pi)^3 = n_\alpha$, with n_α the number density of species α.

We also need the Poisson equation

$$\nabla\cdot\mathbf{E} = 4\pi\sum_\alpha q_\alpha \int f_{\mathbf{p}}^{(\alpha)}\frac{d\mathbf{p}}{(2\pi)^3}, \tag{7}$$

where $\mathbf{E}$ is the wave electric field. We shall assume that the quasineutrality condition $n_i = n_e + Z_d n_d$ is satisfied by the low-frequency motion.

To investigate modulational instability, we must take into account terms up to third order in the fields, including interactions with the virtual waves. The latter are perturbations in the pump electric field at the beat frequency Ω and in general also at the harmonic $2\omega_0$ [27,28]. Thus, for the slowly varying wave envelope $\mathbf{E}_{iL}(\mathbf{r},t)$, where

$$\mathbf{E} = \frac{1}{2}\left[\mathbf{E}_{iL}\exp(-i\omega_{pi}t) + \text{c.c.}\right] \tag{8}$$

of the ion Langmuir pump waves, we have [26]

$$\nabla\cdot\left(i\frac{\partial}{\partial t}\Delta\mathbf{E}_{iL}\right) - \frac{\omega_{pi}}{2r_{De}^2}\nabla\cdot\mathbf{E}_{iL} = \frac{\omega_{pi}}{2}\nabla\cdot\left(\Delta\left[\frac{\delta n_i}{n_{0i}}\mathbf{E}_{iL}\right]\right), \tag{9}$$

where Δ is the Laplacian and we have introduced the effective density modulation δn_i.

The evolution of the effective density modulation δn_i depends on the regime considered. First, if the modulation frequency Ω satisfies $|\Omega| \ll \omega_{pi}$ and $|\mathbf{K}|v_{Ti} \ll |\Omega| \ll |\mathbf{K}|v_{Te}$, where $\mathbf{K}$ is the wave vector of the modulations and $v_{Te(i)} = \sqrt{T_{e(i)}/m_{e(i)}}$ is the electron (ion) thermal speed, the dust particles do not contribute and δn_i is determined by

$$\left[\frac{\partial^2}{\partial t^2} - v_s^2\Delta\right]\frac{\delta n_i}{n_{0i}} = \frac{1}{16\pi n_{0i}m_e}\Delta|\mathbf{E}_{iL}|^2, \tag{10}$$

where $v_s = \sqrt{T_e/m_i}$ is the usual ion sound speed. For this case the earlier treatment [26] of the ion Langmuir wave modualtional instability in a dust-free *two-component* plasma is applicable. Here the slow density perturbations are also associated with the ion modes. We note that (10) is valid only for perturbation wavelengths ($\propto |\mathbf{K}|^{-1}$) much smaller than the inverse electron Debye length r_{De}^{-1}, while the wavelengths ($\propto |\mathbf{k}_0|^{-1}$) of the ion Langmuir waves are larger than r_{De}^{-1}. Therefore, for the situation described by (9) and (10) modulation is possible only when $|\mathbf{K}| \ll |\mathbf{k}_0|$.

A more important case is one that satisfies $|\mathbf{K}|v_{Td} \ll |\Omega| \ll |\mathbf{K}|v_{Ti}$ and $|\mathbf{K}|r_{Di} \ll 1$, where $v_{Td} = \sqrt{T_d/m_d}$ is the thermal speed of the impurity particles. Under these conditions, the low-frequency motion is described by

$$\left[\frac{\partial^2}{\partial t^2} - v_{sd}^2\Delta\right]\frac{\delta n_i}{n_{0i}} = \frac{Z_d^2 n_{0d}}{16\pi n_{0i}^2 m_d}\Delta|\mathbf{E}_{iL}|^2. \quad (11)$$

In this case the slowly varying density perturbations are associated with the dust-acoustic mode. We emphasize that here the magnitude ($|\mathbf{K}|$) of the perturbation wave vector can be both larger and smaller than that ($|\mathbf{k}_0|$) of the pump.

III. DISPERSION RELATION

To find the growth regimes and rates of the modulational instability described by the system (9) and (11), we shall follow standard procedure [21] and present only the essential steps and formulas. We assume that unperturbed (monochromatic pump) wave has an amplitude given by $\mathbf{E}_0$ given by $\mathbf{E}_{iL} = \mathbf{E}_0\delta(\mathbf{k} - \mathbf{k}_0)$. In the lowest approximation, we have the homogeneous and stationary solution δn_{0i} for any $|\mathbf{E}_0|^2$. By renormalizing the background density $n_{0i} \to n_{0i} + \delta n_{0i}$ we can remove the static density perturbation δn_{0i}. Thus, by combining (9) and (11) in the Fourier space, we obtain the dispersion relation

$$1 = \left\{\frac{[\mathbf{k}_0\cdot(\mathbf{K}+\mathbf{k}_0)]^2}{\mathbf{k}_0^2|\mathbf{K}+\mathbf{k}_0|^2\varepsilon_{K+k_0}} + \frac{[\mathbf{k}_0\cdot(\mathbf{K}-\mathbf{k}_0)]^2}{\mathbf{k}_0^2|\mathbf{K}-\mathbf{k}_0|^2\varepsilon_{K-k_0}}\right\}\Sigma|\mathbf{E}_0|^2, \quad (12)$$

where the linear dielectric permittivity is given by

$$\varepsilon_{K\pm k_0} = 1 - \frac{\omega_{pi}^2}{(\Omega\pm\omega_0)^2} + \frac{1}{|\mathbf{K}\pm\mathbf{k}_0|^2 r_{De}^2}, \quad (13)$$

where $\omega_0 \approx \omega_{pi}\left[1 - (1/2|\mathbf{k}_0|^2 r_{De}^2)\right]$ and $\mathbf{k}_0, \mathbf{K}$ are the wave vectors of the ion Langmuir waves and the density modulation, respectively. The parameter Σ is defined by

$$\Sigma = \frac{1}{32\pi n_{0i}T_i}\frac{|\mathbf{K}|^2 v_{sd}^2}{(\Omega^2 - |\mathbf{K}|^2 v_{sd}^2)}. \quad (14)$$

Taking into account the linear dielectric permittivity (13), we shall obtain the dispersion relations for the limits $|\mathbf{K}| \gg |\mathbf{k}_0|$ and $|\mathbf{K}| \ll |\mathbf{k}_0|$. For $|\mathbf{K}| \gg |\mathbf{k}_0|$, Eq. (12) takes the form

$$1 = \left[8\frac{\Omega}{\omega_{pi}}\frac{|\mathbf{k}_0|}{|\mathbf{K}|}\cos\Theta\sin^2\Theta + \frac{2\cos^2\Theta}{|\mathbf{k}_0|^2 r_{De}^2}\right] \times\left[4\left(\frac{\Omega}{\omega_{pi}}\right)^2 - 8\frac{\Omega}{\omega_{pi}}\frac{|\mathbf{k}_0|\cos\Theta}{|\mathbf{K}|^3 r_{De}^2} - \frac{1}{|\mathbf{k}_0|^4 r_{De}^4}\right]^{-1}\Sigma|\mathbf{E}_0|^2, \quad (15)$$

where Θ is the angle between $\mathbf{K}$ and $\mathbf{k}_0$. In the opposite limit $|\mathbf{K}| \ll |\mathbf{k}_0|$, we find

$$1 = \left[8\frac{\Omega}{\omega_{pi}}\frac{|\mathbf{k}_0|}{|\mathbf{K}|}\cos\Theta\sin^2\Theta - \frac{2\left(4\cos^2\Theta - 1\right)}{|\mathbf{K}|^2 r_{De}^2}\right] \times\left[4\left(\frac{\Omega}{\omega_{pi}}\right)^2\left(\frac{|\mathbf{k}_0|}{|\mathbf{K}|}\right)^4 - 8\frac{\Omega}{\omega_{pi}}\frac{|\mathbf{k}_0|\cos\Theta}{|\mathbf{K}|^3 r_{De}^2} + \frac{4\cos^2\Theta}{|\mathbf{k}_0|^2|\mathbf{K}|^2 r_{De}^4}\right]^{-1}\Sigma|\mathbf{E}_0|^2. \quad (16)$$

We shall obtain in the following section solutions of the dispersion relations (15) and (16).

IV. GROWTH RATES

A. Short-wavelength ($|\mathbf{K}| \gg |\mathbf{k}_0|$) modulations

When the wavelength of the pump ion Langmuir wave is long ($|\mathbf{K}| \gg |\mathbf{k}_0|$), the dispersion equation takes the form (15). We let $|\cos\Theta| \sim |\sin\Theta| \sim 1$, so that interactions with the largest growth rates are represented. Interactions corresponding to $\mathbf{K} \parallel \mathbf{k}_0$ and $\mathbf{K} \perp \mathbf{k}_0$ are not so important since for these the growth rates are much smaller.

We also assume

$$\frac{\Omega}{\omega_{pi}} \gg \frac{1}{|\mathbf{k}_0|^2 r_{De}^2}\frac{|\mathbf{K}|}{|\mathbf{k}_0|}, \quad (17)$$

so that (15) reduces to

$$1 = 2\frac{\omega_{pi}}{\Omega}\frac{|\mathbf{k}_0|}{|\mathbf{K}|}\cos\Theta\sin^2\Theta\Sigma|\mathbf{E}_0|^2, \quad (18)$$

which corresponds to the case with the maximum growth rate for $|\mathbf{K}| \gg |\mathbf{k}_0|$. Here, only supersonic ($\gamma \gg |\mathbf{K}|v_{sd}$) development of the modulational instability is possible. The growth rate of the instability is

$$\gamma \sim \left(|\mathbf{K}||\mathbf{k}_0|v_{sd}^2\omega_{pi}W\right)^{\frac{1}{3}}, \quad (19)$$

where $W = |\mathbf{E}_0|^2/16\pi n_{0i}T_i$. The growth rate can have the maximum value given by

$$\frac{\gamma_{\max}}{\omega_{pi}} \sim \min\left\{|\mathbf{k}_0|^2 r_{De}^2 \left(\frac{\alpha T_i W}{T_e}\right)^{\frac{1}{2}}, (|\mathbf{k}_0| r_{De})^{\frac{5}{6}} \left(\alpha \frac{T_i}{T_e}\right)^{\frac{1}{4}} W^{\frac{1}{2}}, \left(\frac{T_i}{T_e}\right)^{\frac{1}{6}} (|\mathbf{k}_0| r_{De} \alpha W)^{\frac{1}{3}}\right\}, \quad (20)$$

where $\alpha = m_i Z_d / m_d$. Equation (20) follows from the conditions (17), $\gamma \gg |\mathbf{K}| v_{sd}$, as well as the inequality $|\mathbf{K}| r_{Di} \ll 1$. The maximum growth rate can be realized if

$$W \gg \max\left\{\frac{1}{|\mathbf{k}_0|^8 r_{De}^8} \frac{T_e}{\alpha T_i}, |\mathbf{k}_0| r_{De} \left(\frac{\alpha T_i}{T_e}\right)^{\frac{1}{2}} \frac{|\mathbf{K}|^2}{|\mathbf{k}_0|^2}\right\}. \quad (21)$$

We consider next the case

$$\frac{1}{|\mathbf{k}_0|^2 r_{De}^2} \ll \frac{\Omega}{\omega_{pi}} \ll \frac{1}{|\mathbf{k}_0|^2 r_{De}^2} \frac{|\mathbf{K}|}{|\mathbf{k}_0|}. \quad (22)$$

The corresponding dispersion relation is

$$1 = \frac{\omega_{pi}^2}{\Omega^2} \frac{\cos^2\Theta}{2|\mathbf{k}_0|^2 r_{De}^2} \Sigma |\mathbf{E}_0|^2. \quad (23)$$

In this case both supersonic ($\gamma > |\mathbf{K}| v_{sd}$) and subsonic ($\gamma < |\mathbf{K}| v_{sd}$) growths are possible. For the fast growth limit the instability develops with the rate

$$\gamma \sim \omega_{pi} \left(\frac{|\mathbf{K}|^2}{|\mathbf{k}_0|^2} \frac{\alpha T_i}{4T_e} W\right)^{\frac{1}{4}}, \quad (24)$$

for which the pump amplitude must satisfy

$$\frac{|\mathbf{k}_0|^2}{|\mathbf{K}|^2} \ll |\mathbf{k}_0|^8 r_{De}^8 \frac{\alpha T_i}{4T_e} W \ll \frac{|\mathbf{K}|^2}{|\mathbf{k}_0|^2} \quad (25)$$

and

$$\alpha \ll \frac{T_e}{4T_i} \frac{W}{|\mathbf{k}_0|^2 |\mathbf{K}|^2 r_{De}^4} \ll \frac{1}{\alpha}. \quad (26)$$

In the subsonic limit we find

$$\gamma \sim \frac{\omega_{pi}}{2|\mathbf{k}_0| r_{De}} W^{\frac{1}{2}}, \quad (27)$$

which can occur if

$$\frac{T_d}{T_e} \frac{m_i}{m_d} \ll \frac{W}{4|\mathbf{k}_0|^2 |\mathbf{K}|^2 r_{De}^4} \ll \frac{\alpha T_i}{T_e} \quad (28)$$

and

$$1 \ll \frac{1}{4} |\mathbf{k}_0|^2 r_{De}^2 W \ll \frac{|\mathbf{K}|^2}{|\mathbf{k}_0|^2}. \quad (29)$$

Finally, for the case of very slow modulation satisfying

$$\frac{\Omega}{\omega_{pi}} \ll \frac{1}{|\mathbf{k}_0|^2 r_{De}^2}, \quad (30)$$

the dispersion equation (15) reduces to

$$1 = -2|\mathbf{k}_0|^2 r_{De}^2 \cos^2\Theta \Sigma |\mathbf{E}_0|^2. \quad (31)$$

The corresponding growth rate of the modulational instability is

$$\gamma = |\mathbf{K}| v_{sd} \left(|\mathbf{k}_0|^2 r_{De}^2 \cos^2\Theta W - 1\right)^{\frac{1}{2}}, \quad (32)$$

which occurs for

$$1 < |\mathbf{k}_0|^2 r_{De}^2 W \ll \min\left\{\frac{1}{\alpha}, \frac{1}{|\mathbf{k}_0|^4 |\mathbf{K}|^2 r_{De}^6} \frac{T_e}{\alpha T_i}\right\}. \quad (33)$$

Thus we see that modulational instability of the ion Langmuir waves with wavelengths larger than those of the pump wave can appear in a dusty plasma. Since these waves can interact with the thermal ions, energy transfer to the latter becomes possible. It follows that in a dusty plasma the bulk plasma ions can be heated by the ion Langmuir waves. The corresponding rate of ion heating depends on the parameters of the plasma as well as the pump wave. For example, if

$$\left(\frac{T_i}{T_e}\right)^{\frac{1}{6}} (|\mathbf{k}_0| r_{De} \alpha W)^{\frac{1}{3}} \le \min\left\{|\mathbf{k}_0|^2 r_{De}^2 \left(\frac{\alpha T_i}{T_e} W\right)^{\frac{1}{2}}, (|\mathbf{k}_0| r_{De})^{\frac{5}{6}} \left(\frac{\alpha T_i}{T_e}\right)^{\frac{1}{4}} W^{\frac{1}{2}}\right\}, \quad (34)$$

then ion Langmuir wave modulations with $|\mathbf{K}|r_{Di} \sim 1$ (i.e., those which are resonant with the thermal ions) grow at the rate (19). The rate of bulk ions heating can be estimated to be

$$\frac{1}{T_i}\frac{dT_i}{dt} \sim \omega_{pi}\left(\frac{\alpha|\mathbf{k}_0|r_{De}|\mathbf{E}_0|^8}{n_{0i}^4 T_i^{7/2} T_e^{1/2}}\right)^{\frac{1}{3}}. \quad (35)$$

We emphasize that the possibility of heating the bulk plasma ions is the result of nonlinear coupling between finite-amplitude ion Langmuir waves and the dust-acoustic perturbations. Modulational processes in which the low-frequency perturbations are associated with the ordinary ion-sound perturbations do not result in such ion heating because the limit $|\mathbf{K}| \gg |\mathbf{k}_0|$ does not occur.

B. Long-wavelength modulations

We now consider the case $|\mathbf{K}| \ll |\mathbf{k}_0|$, for which the dispersion relation (22) is valid. Again we let $|\cos\Theta| \sim |\sin\Theta| \sim 1$ and investigate three regimes of Ω/ω_{pi}. For the regime

$$\frac{\Omega}{\omega_{pi}} \gg \frac{1}{|\mathbf{k}_0|^2 r_{De}^2}\frac{|\mathbf{k}_0|}{|\mathbf{K}|}, \quad (36)$$

the dispersion relation (22) reduces to

$$1 = 2\frac{\omega_{pi}}{\Omega}\frac{|\mathbf{K}|^3}{|\mathbf{k}_0|^3}\cos\Theta\sin^2\Theta\Sigma|\mathbf{E}_0|^2. \quad (37)$$

Here, there exist unstable solutions only in the supersonic ($\gamma > |\mathbf{K}|v_{sd}$) regime. The growth rate of the instability is

$$\gamma \sim \frac{|\mathbf{K}|}{|\mathbf{k}_0|}\left(|\mathbf{K}|^2 v_{sd}^2 \omega_{pi} W\right)^{\frac{1}{3}}, \quad (38)$$

which is much smaller than that of (19) because of the condition $|\mathbf{K}| \ll |\mathbf{k}_0|$ here. The above instability occurs when

$$W \gg \max\left\{\frac{1}{|\mathbf{k}_0|^8 r_{De}^8}\frac{T_e}{\alpha T_i},\ |\mathbf{k}_0|r_{De}\frac{|\mathbf{k}_0|^2}{|\mathbf{K}|^2}\left(\frac{\alpha T_i}{T_e}\right)^{\frac{1}{2}}\right\}. \quad (39)$$

Next, for the regime

$$\frac{1}{|\mathbf{k}_0|^2 r_{De}^2}\frac{|\mathbf{K}|}{|\mathbf{k}_0|} \ll \frac{\Omega}{\omega_{pi}} \ll \frac{1}{|\mathbf{k}_0|^2 r_{De}^2}\frac{|\mathbf{k}_0|}{|\mathbf{K}|}, \quad (40)$$

we find the dispersion equation

$$1 = \frac{\omega_{pi}^2}{\Omega^2}\frac{\left(1-4\cos^2\Theta\right)}{2|\mathbf{k}_0|^2 r_{De}^2}\frac{|\mathbf{K}|^2}{|\mathbf{k}_0|^2}\Sigma|\mathbf{E}_0|^2. \quad (41)$$

For supersonic ($\gamma \gg |\mathbf{K}|v_{sd}$) growth, the instability develops with the rate

$$\gamma \sim \omega_{pi}\left(\frac{|\mathbf{K}|^4}{|\mathbf{k}_0|^4}\frac{\alpha T_i}{4T_e}W\right)^{\frac{1}{4}}, \quad (42)$$

which occurs for

$$1 \ll \frac{|\mathbf{k}_0|^8 r_{De}^8 \alpha T_i W}{4T_e} \ll \frac{|\mathbf{k}_0|^8}{|\mathbf{K}|^8} \quad (43)$$

and

$$\alpha \ll \frac{T_e W}{T_i|\mathbf{k}_0|^4 r_{De}^4} \ll \frac{1}{\alpha}. \quad (44)$$

Here subsonic ($\gamma \ll |\mathbf{K}|v_{sd}$) growth is possible only for $\cos^2\Theta < 1/4$. The rate of the instability is

$$\gamma \sim \frac{\omega_{pi}}{2|\mathbf{k}_0|r_{De}}\frac{|\mathbf{K}|}{|\mathbf{k}_0|}W^{\frac{1}{2}}, \quad (45)$$

which occurs for

$$\frac{m_i}{m_d}\frac{T_d}{T_e} \ll \frac{W}{4|\mathbf{k}_0|^4 r_{De}^4} \ll \frac{\alpha T_i}{T_e} \quad (46)$$

and

$$1 \ll \frac{1}{4}|\mathbf{k}_0|^2 r_{De}^2 W \ll \frac{|\mathbf{k}_0|^4}{|\mathbf{K}|^4}. \quad (47)$$

Finally, we consider the regime

$$\frac{\Omega}{\omega_{pi}} \ll \frac{1}{|\mathbf{k}_0|^2 r_{De}^2}\frac{|\mathbf{K}|}{|\mathbf{k}_0|}. \quad (48)$$

Here the dispersion relation takes the form

$$1 = -2|\mathbf{k}_0|^2 r_{De}^2 \Psi\Sigma|\mathbf{E}_0|^2, \quad (49)$$

where $\Psi = (4\cos^2\Theta - 1)/4\cos^2\Theta$. The instability can develop only for $\cos^2\Theta > 1/4$. The growth rate is

$$\gamma = |\mathbf{K}|v_{sd}\left(|\mathbf{k}_0|^2 r_{De}^2 \Psi W - 1\right)^{\frac{1}{2}}, \quad (50)$$

which occurs for

$$1 < |\mathbf{k}_0|^2 r_{De}^2 \Psi W \ll \min\left\{\frac{1}{\alpha},\ \frac{1}{|\mathbf{k}_0|^6 r_{De}^6}\frac{T_e}{\alpha T_i}\right\}. \quad (51)$$

Since $\Psi < 1$, the necessary condition for the instability is

$$W > \frac{1}{|\mathbf{k}_0|^2 r_{De}^2},$$

which is coincident with the left inequality of (33).

V. ION LANGMUIR SOLITONS

Here we consider one-dimensional localized solutions of the set (9) and (11). All vectors are assumed to be parallel to the x axis and the following dimensionless variables are defined:

$$\tau = \left(\frac{v_{sd}^2\omega_{pi}}{2r_{De}^2}\right)^{\frac{1}{3}} t,$$

$$\xi = \left(\frac{\omega_{pi}}{2r_{De}^2 v_{sd}}\right)^{\frac{1}{3}} x,$$

$$\nu = \left(\frac{v_s}{2v_{sd}}\right)^{\frac{2}{3}} \left(\frac{\delta n_i}{n_{0i}}\right),$$

and

$$\varepsilon = \frac{1}{v_{sd}}\left(\frac{v_s}{2v_{sd}}\right)^{\frac{1}{3}} \left(\frac{Z_d^2 n_{0d}}{16\pi n_{0i}^2 m_d}\right)^{\frac{1}{2}} |\mathbf{E}_{iL}|. \quad (52)$$

Equations (9) and (11) can be written as

$$\frac{\partial^2}{\partial\xi^2}\left(i\frac{\partial\varepsilon}{\partial\tau} - \nu\varepsilon\right) = \varepsilon \quad (53)$$

and

$$\frac{\partial^2\nu}{\partial\tau^2} - \frac{\partial^2\nu}{\partial\xi^2} = \frac{\partial^2|\varepsilon|^2}{\partial\xi^2}. \quad (54)$$

We seek localized solutions moving with a constant speed u, such that $\varepsilon = \varepsilon(\tau,\xi;\zeta)$ and $\nu = \nu(\zeta)$, where $\zeta = \xi - u\tau$. In this case, one obtains, from (53) and (54),

$$\frac{\partial^2}{\partial\xi^2}\left(i\frac{\partial\varepsilon}{\partial\tau} + \frac{|\varepsilon|^2\varepsilon}{1-u^2}\right) = \varepsilon, \quad (55)$$

which is well known for media with inverse dispersion and has been investigated earlier [29–31].

Assuming $\varepsilon = \psi(\zeta)\exp(iS)$, where $\partial_\tau S = -\Omega_0 - ku$ and $\partial_\xi S = k(\zeta)$, and separating the real and imaginary parts of (55), one finds

$$\frac{1}{\psi^2}\left(\frac{d}{d\zeta}\psi^2 R\right)^2 + M^2 = 2\psi^2 R + D,$$

$$\frac{dM}{d\zeta} = \frac{k}{\psi}\frac{d}{d\zeta}\psi^2 R, \quad (56)$$

$$u\frac{d^2\psi}{d\zeta^2} = \psi k V + M,$$

where D is a constant, Ω_0 is the nonlinear frequency shift, and we have defined $V = ku + \Omega_0 + \psi^2/(1-u^2)$ and $R = ku + \Omega_0/2 + 3\psi^2/4(1-u^2)$. The necessary asymptotic behavior ($\psi \to 0$ when $\zeta \to \pm\infty$) for localized solutions can be realized only if $u \ll \Omega_0^{3/2}$. In that case, Eq. (56) can be written as

$$M = u\frac{d^2\psi}{d\zeta^2} - \psi k V,$$

$$k = \left(u\frac{d^3\psi}{d\zeta^3} - \frac{d}{d\zeta}\psi k V\right)\left[\frac{d}{d\zeta}\psi\left(\Omega_0 + \frac{\psi^2}{1-u^2}\right)\right]^{-1}, \quad (57)$$

and

$$\left(1-\frac{\psi^2}{\psi_c^2}\right)\left(\frac{d\psi}{d\zeta}\right)^2 + \frac{u^2}{\Omega_0^2}\left[\frac{d^2\psi}{d\zeta^2} - \frac{\psi k\Omega_0}{u}\left(1-\frac{\psi^2}{3\psi_c^2}\right)\right] = \frac{\psi^2}{\Omega_0}\left(1-\frac{\psi^2}{2\psi_c^2}\right), \quad (58)$$

where we have set $D = 0$ and defined $\psi_c^2 = \Omega_0(u^2-1)/3$. Finite solutions of (58) exist only when $\Omega_0 > 0$ and $u > 1$ (so that $\psi_c^2 > 0$). In this case the localized solution contains a singular point at $\psi = \psi_c$, where the derivatives are discontinuous.

We can visualize the solution as follows. For $\psi \ll \psi_c$, one can omit the second term in (58) and obtain

$$\psi = \psi' \exp(-|\zeta|/\zeta_*), \quad (59)$$

where $\zeta_* = \Omega_0^{1/2}$, and (59) is valid for $|\psi/\psi_c - 1| > [u(u^2-1)/\sqrt{2}\Omega_0^{3/2}]^{\frac{1}{3}}$.

On the other hand, near the soliton apex ($\psi \approx \psi_c$) one can neglect the first derivative in (58) if $k \sim k_0 = C/\sqrt{\Omega_0}$, where C is a constant, and obtain

$$\psi \sim \exp(-|\zeta|/\zeta_{**}), \quad (60)$$

where $\zeta_{**} = (\sqrt{\Omega_0}/u)^{-\frac{1}{2}}(\sqrt{2}/2 + 2C/3)^{-\frac{1}{4}}$. From the equation for k in (57) one finds $C = \sqrt{2}$. Comparing (59) and (60) and taking into account condition $u \ll \Omega_0^{3/2}$, we see that $\zeta_{**} < \zeta_*$. That is, there are two characteristic widths in the solution. They are given by $\zeta_{**} \sim [u(u^2-1)^{1/2}/\psi_c]^{1/2}$ near the apex and $\zeta_* = \Omega_0^{1/2} = \psi_c[3/(u^2-1)]^{\frac{1}{2}}$ at the edges, so that the solution varies more rapidly near the apex. A typical localized solution of (58) is presented in Fig. 1.

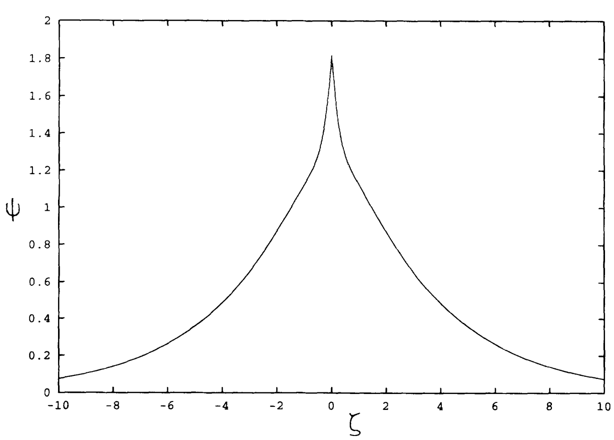

FIG. 1. Profile of the solitary ion Langmuir wave envelope $\psi(\zeta)$ for $\Omega_0 = 10$ and $u = 1.41$.

It can be shown that near the apex, the soliton phase $\int_0^\zeta k(\zeta')d\zeta'$ remains finite. In fact, there are $\sim u/\Omega_0^{3/2}$ ($\ll 1$) phase oscillations, and in the edge regions the number of phase oscillations is $\sim (u/\Omega_0^{3/2})^{1/2}$.

A physically important feature of the solution of (55) is that harmonics with large $|\mathbf{K}|$ are present. These short-wavelength harmonics interact strongly with the thermal ions and are thus heavily damped. Since all the harmonics in the soliton are correlated in amplitude and phase, the entire soliton will also be damped, leading to the heating of the bulk plasma ions.

VI. SUMMARY

To summarize, we have investigated the nonlinear interaction between finite amplitude short-wavelength ion-acoustic, or the ion Langmuir, waves and dust-acoustic perturbations in a uniform unmagnetized impurity-containing plasma. We obtained a pair of nonlinear equations governing the self-consistent interaction. The equations are then used to study the development of the modulational instability as well as the evolution of the nonlinear ion Langmuir waves. The modulational instability can develop at all angles between the wave vectors of the pump wave and the low-frequency modulations. A feature of the modulational instability of ion Langmuir waves arising from interaction with dust-acoustic perturbations is the possibility of unstable perturbation wavelengths larger than those of the pump wave. This feature distinguishes the present modulational instability from others involving ion-acoustic (or the short-wavelength ion Langmuir) waves in that here resonant interaction with thermal ions can occur. Thus the bulk of the plasma ions can be heated.

We have also studied the existence and properties of ion Langmuir solitons, which could be formed as a result of the interaction. The equation governing the nonlinear evolution of the modulational instability is typical for media with inverse, or negative, dispersion. Similar spiky envelope solitons were found for other media with inverse dispersion [31]. A unique feature of the ion Langmuir solitons is the presence in its Fourier spectrum of harmonics with large wave vectors. These harmonics can strongly interact with the thermal ions and result in the damping of the solitons and the heating of the thermal ions in a dusty plasma. On the other hand, for applications in dusty plasmas, one may have to take into account the dust charging process using an appropriate model [8,16,17] for the latter.

Thus, for plasmas containing heavy impurity or dust particles, the modulational instability of finite-amplitude short-wavelength ion-acoustic waves can lead to the heating of the bulk plasma ions at both the initial and final stages of its evolutions. This behavior can be useful in explaining ion-heating phenomena in space and other plasmas, as well as in the development of methods for ion heating in fusion-related plasmas.

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